1. Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\frac{dy}{dx}$.

$\frac{dy}{dx} = 4x^3 + \frac{1}{3}x^{-\frac{2}{3}}$

2. (a) Expand and simplify $(7 + \sqrt{5})(3 - \sqrt{5})$.

(b) Express $\dfrac{7+\sqrt{5}}{3+\sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers.

(3)

a) $21 + 3\sqrt{5} - 7\sqrt{5} - 5 = 16 - 4\sqrt{5}$

b) $\dfrac{7+3\sqrt{5}}{3+\sqrt{5}} \times \dfrac{(3-\sqrt{5})}{(3-\sqrt{5})} = \dfrac{16-4\sqrt{5}}{9-5}$ (a)

$= \dfrac{16}{4} - \dfrac{4\sqrt{5}}{4} = \underline{4 - \sqrt{5}}$ $a = 4$

$b = -1$

3. The line l_1 has equation $3x + 5y - 2 = 0$

(a) Find the gradient of l_1.

The line l_2 is perpendicular to l_1 and passes through the point (3, 1).

(b) Find the equation of l_2 in the form $y = mx + c$, where m and c are constants.

(3)

(a) $5y = -3x + 2 \Rightarrow y = -\frac{3}{5}x + \frac{2}{5}$ $\quad \underline{m = -\frac{3}{5}}$

(b) l_2 perp $\Rightarrow m_{l_2} = +\frac{5}{3}$ $\quad x_1 = 3 \quad y_1 = 1$

$y - 1 = \frac{5}{3}(x - 3) \Rightarrow 3y - 3 = 5x - 15$

$3y = 5x - 12 \Rightarrow \underline{y = \frac{5}{3}x - 4}$

$x\sqrt{x} = x^1$

4. $$\frac{dy}{dx} = 5x^{-\frac{1}{2}} + x\sqrt{x}, \qquad x > 0$$ $= x^{\frac{3}{2}}$

Given that $y = 35$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

(7)

$$y = \frac{5x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + \frac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + c$$

$$y = 10x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + c \qquad (4, 35)$$

$$35 = 10 \times 2 + \frac{2}{5} \times 32 + c$$

$$35 = 20 + 12.8 + c \qquad c = 2.2 = \frac{11}{5}$$

$$y = 10\sqrt{x} + \frac{2}{5}\sqrt{x^5} + \frac{11}{5}$$

6. Solve the simultaneous equations

$$y - 3x + 2 = 0$$

$$y^2 - x - 6x^2 = 0$$

(7)

$y = 3x - 2 \qquad y^2 = 9x^2 - 12x + 4$

$(9x^2 - 12x + 4) - x - 6x^2 = 0$

$3x^2 - 13x + 4 = 0$

$(3x - 1)(x - 4) = 0$

$x = \frac{1}{3} \qquad x = 4$

$x = \frac{1}{3} \quad y = 3(\frac{1}{3}) - 2 = -1 \qquad \underline{(\frac{1}{3}, -1)}$

$x = 4 \quad y = 3(4) - 2 = 10 \qquad \underline{(4, 10)}$

6. The curve C has equation

$$y = \frac{(x+3)(x-8)}{x}, \quad x > 0$$

(a) Find $\frac{dy}{dx}$ in its simplest form.

(4)

(b) Find an equation of the tangent to C at the point where $x = 2$

(4)

(a) $y = \frac{x^2}{x} - \frac{5x}{x} - \frac{24}{x} \Rightarrow y = x - 5 - 24x^{-1}$

$\frac{dy}{dx} = 1 + 24x^{-2}$

(b) $x = 2 \Rightarrow m_t = 1 + 24 \times 2^{-2}$ $\quad 2^{-2} = \frac{1}{4}$

$m_t = 7$

$x = 2 \quad y = \frac{(5)(-6)}{2} = -15 \quad x_1 = 2 \quad y_1 = -15$

$y + 15 = 7(x-2) \Rightarrow y + 15 = 7x - 14$

$\underline{y = 7x - 29}$

7. Jill gave money to a charity over a 20-year period, from Year 1 to Year 20 … She gave £150 in Year 1, £160 in Year 2, £170 in Year 3, and so on, so that the a… of money she gave each year formed an arithmetic sequence.

(a) Find the amount of money she gave in Year 10.

(2)

(b) Calculate the total amount of money she gave over the 20-year period.

(3)

Kevin also gave money to the charity over the same 20-year period.

He gave £A in Year 1 and the amounts of money he gave each year increased, forming an arithmetic sequence with common difference £30.

The total amount of money that Kevin gave over the 20-year period was **twice** the total amount of money that Jill gave.

(c) Calculate the value of A.

(4)

(a) $u_{10} = a + 9d = £150 + 9 \times £10 = \underline{£240}$

(b) $S_{20} = 10(300 + 19 \times 10) = 10 \times (490) = \underline{£4900}$

(c) Kevin $S_{20} = £9800$

$9800 = 10(2A + 19 \times 30)$

$980 = 2A + 570 \Rightarrow 2A = 410 \quad \underline{A = £205}$

8.

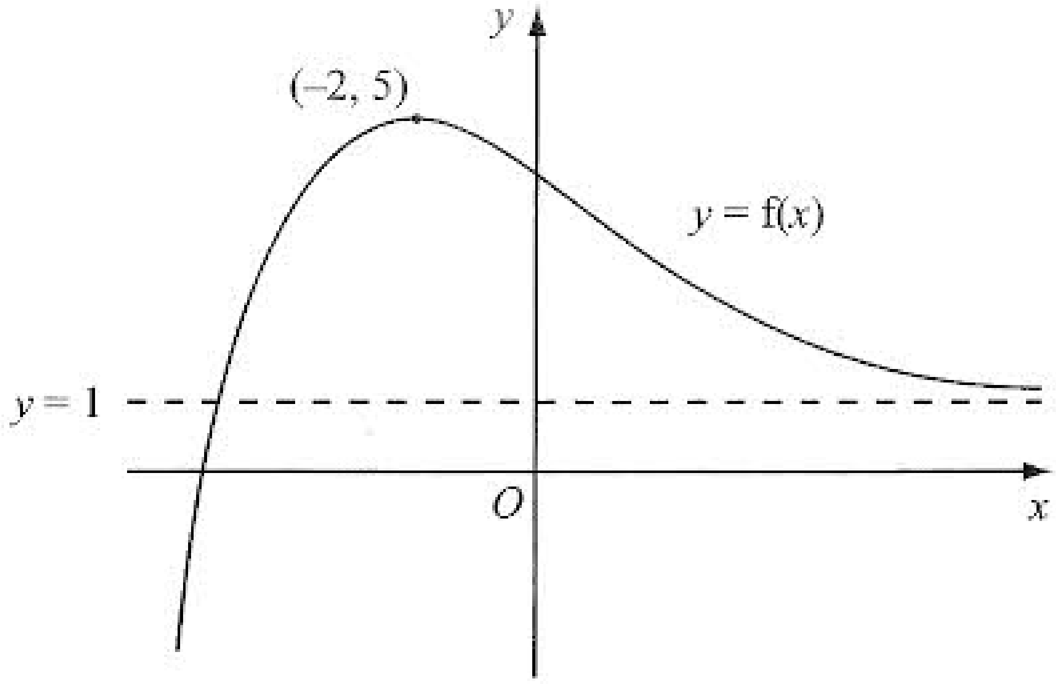


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$.

The curve has a maximum point $(-2, 5)$ and an asymptote $y = 1$, as shown in Figure 1.

On separate diagrams, sketch the curve with equation

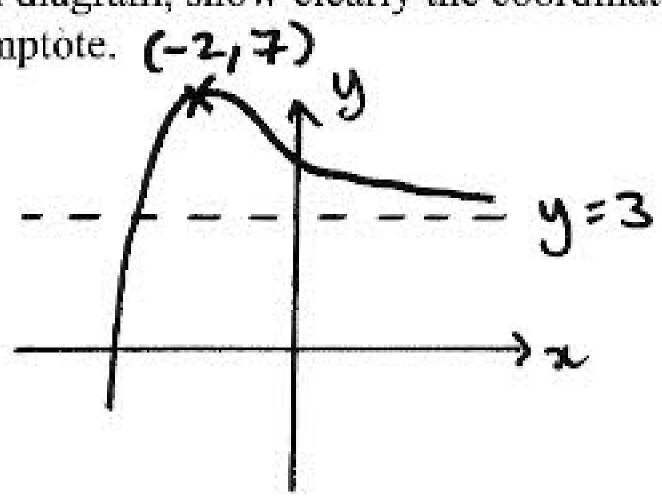
(a) $y = f(x) + 2$ $+2 \uparrow$ **(2)**

(b) $y = 4f(x)$ $\updownarrow \times 4$ **(2)**

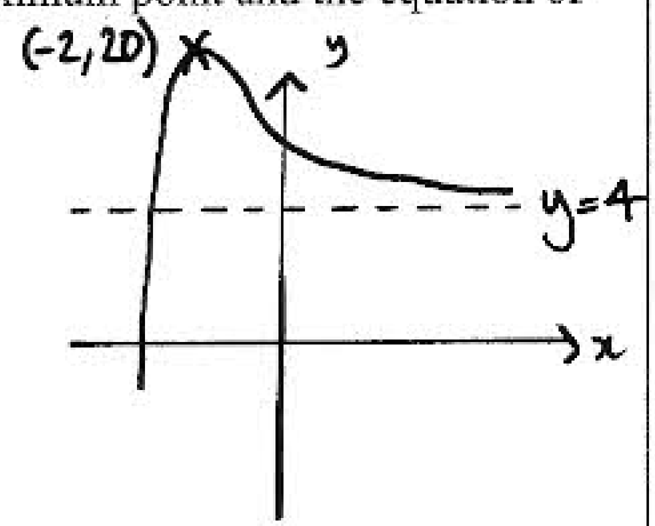
(c) $y = f(x + 1)$ $\xleftarrow[1]{}$ **(3)**

On each diagram, show clearly the coordinates of the maximum point and the equation of the asymptote.

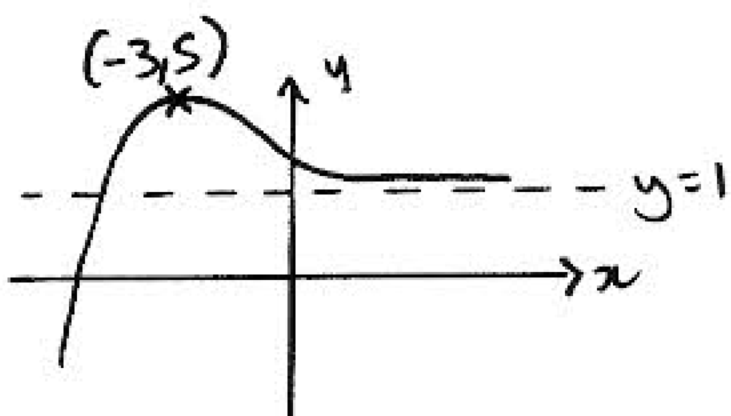
a)



b)



c)



9. (a) Factorise completely $x^3 - 4x$ $= x(x^2-4) = x(x-2)($

0 $\uparrow$ cross $\qquad 2 \quad 2$ touch

(b) Sketch the curve C with equation

$$y = x^3 - 4x,$$ $+\infty \times +\infty \times +\infty = +\infty \nearrow$

showing the coordinates of the points at which the curve meets the x-axis.

(3)

The point A with x-coordinate -1 and the point B with x-coordinate 3 lie on the curve C.

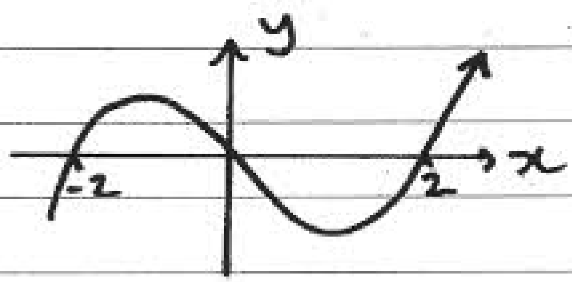
(c) Find an equation of the line which passes through A and B, giving your answer in the form $y = mx + c$, where m and c are constants.

(5)

(d) Show that the length of AB is $k\sqrt{10}$, where k is a constant to be found.

(2)

b)



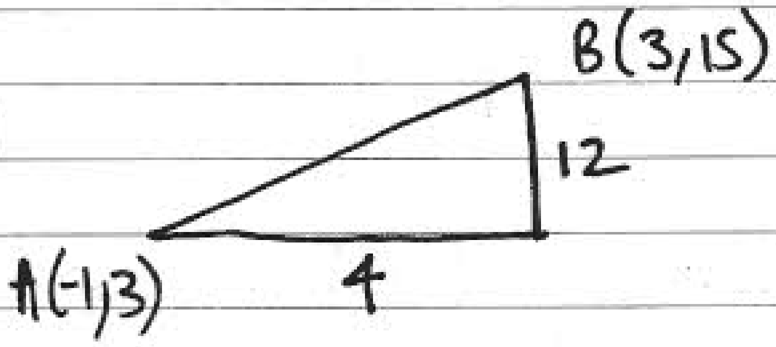
c) $x = -1 \quad y = (-1)^3 - 4(-1) = -1 + 4 = 3 \quad (-1,3)A$

$x = 3 \quad y = (3)^3 - 4(3) = 27 - 12 = 15 \quad (3,15)B$

$m_{AB} = \dfrac{15-3}{3--1} = \dfrac{12}{4} = 3$

using A, $y - 3 = 3(x+1) \Rightarrow y - 3 = 3x + 3 \Rightarrow \underline{y = 3x + 6}$

d)



$AB^2 = 4^2 + 12^2 = 16 + 144$

$AB = \sqrt{160} = \sqrt{16}\sqrt{10}$

$AB = \underline{4\sqrt{10}}$

10. $\quad f(x) = x^2 + 4kx + (3 + 11k),\quad$ where k is a constant.

(a) Express f(x) in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k.
(3)

Given that the equation f(x) = 0 has no real roots,

(b) find the set of possible values of k.
(4)

Given that $k = 1$,

(c) sketch the graph of $y = f(x)$, showing the coordinates of any point at which the graph crosses a coordinate axis.
(3)

(a) $f(x) = (x+2k)^2 - 4k^2 + (3+11k)$

$= (x+2k)^2 - 4k^2 + 11k + 3$

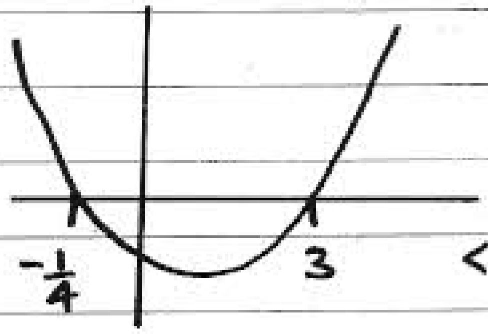
(b) $b^2 - 4ac < 0$

$(4k^2) - 4(1)(3+11k) < 0$

$16k^2 - 44k - 12 < 0 \Rightarrow 4k^2 - 11k - 3 < 0$

$(4k+1)(k-3) < 0$

$-\frac{1}{4} \qquad 3$



$-\frac{1}{4} < k < 3$

(c) $k = 1 \quad y = (x+2)^2 - 4 + 11 + 3 \quad y = (x+2)^2 + 10$

$x = 0 \quad y = 14$

TP$(-2, 10)$